Transitional Flow on Axial Turbomachine Blading

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Correlations of data from constant-pressure flows are shown to seriously overestimate the transition length on axial turbomachine blades in regions of positive pressure gradient. The differences largely can be ascribed to the influence of the pressure gradient on the mechanism of laminar-turbulent transition. A hypothesis of continuous breakdown of laminar instability waves, together with an estimate of the dominant disturbance frequency, is introduced to model the minimum possible length of transitional flow in terms of local boundary-layer parameters. This model should be preferable to a point transition assumption for predicting turbomachine blade flows. In practice, the transition length most probably will lie between the predictions of the minimum length model and those of constant-pressure flow correlations.

Nomenclature

c = chord length
C_r = phase velocity
G = transition length parameter, Eq. (10)
L_T = transition length, $= x_T - x_t$
Re = Reynolds number, $= u_e x/\nu$
Re_c = chord Reynolds number, $u_\infty c/\nu$
Re_{L_T} = transition length Reynolds number, $= u_e L_T/\nu$
Re_{δ^*} = boundary-layer Reynolds number, $= u_e \delta^*/\nu$
Re_λ = transition length Reynolds number, $= u_e \lambda/\nu$
s = peripheral blade spacing
t = time
Tu = freestream turbulence level, %
u_e = local freestream velocity
u_∞ = vector mean velocity for a cascade and freestream velocity for isolated aerofoil
U_1 = local freestream velocity
z = spanwise distance
α = half-angle of turbulent spot envelope
β = Falkner-Skan pressure gradient parameter
γ = intermittency of turbulence
δ = boundary-layer thickness
δ^* = displacement thickness
λ = distance between 0.25 and 0.75 intermittency points
Λ_x = wavelength of Tollmien-Schlichting instability
ν = kinematic viscosity
ξ = dimensionless position, $= (x - x_t)/\lambda$
ω = circular frequency, rad $\cdot$ s^{-1}

Subscripts

l = laminar
t = value at start of transition region ($\gamma = 0$)
T = value at end of transition region ($\gamma = 0.99$)

I. Introduction

THE computation of viscous flow around airfoils is critically dependent on the modeling of laminar-turbulent transition in the airfoil boundary layers. Current calculation methods are capable of producing realistic loss estimates when experimental transition data are employed. However, it remains difficult to predict the onset and extent of transitional flow a priori, and this is currently a major source of uncertainty in airfoil flow calculations.

This paper is principally concerned with specifying transition behavior for calculations of the time-averaged flow state. In this context, it is necessary to specify only a time-averaged value of turbulent intermittency at a given point in space; periodic variations of intermittency (which would have to be specified for a full nonsteady boundary-layer calculation) are ignored. The form of time averaging used is dependent on the nature of flow unsteadiness: conventional Reynolds averaging of the Navier-Stokes equations is sufficient for flows with purely random freestream disturbances; a further averaging over a multiple of the shaft rotation period is required for turbomachinery flows that also involve periodic disturbances due to effects such as inlet distortions and wake chopping.

For aircraft wings operating at chord Reynolds numbers Re_c of 10^6–10^7, the extent of transitional flow can be as small as 5% chord.[1] In such cases, a point transition model often has been employed for simplicity, although it introduces the additional complication of specifying a sudden jump in various boundary-layer parameters from the laminar to the turbulent state.

Axial turbomachine blades operate at much lower Reynolds numbers, with Re_c typically of 10^5–10^6 and relatively greater lengths of transitional flow. For example, Fig. 1 shows the variation with incidence of the transition zone in decelerating flow on the suction surface of an axial compressor blade section operated in a two-dimensional cascade[2] at $Re_c = 2 \times 10^5$ and a freestream turbulence level of 0.2%. The length of the transition zone (obtained from qualitative hot-wire observations) is around 10% of chord, which appears to be fairly typical on aerofoils in cascade where the transition occurs in decelerating flow. The results of Hodson[3] and Turner[4] with freestream turbulence around 0.5% are quite similar.

For turbomachine blade flows, a point transition model becomes increasingly inadequate, especially where laminar separation bubbles are present on the blade surface. The extent of transitional flow in a separation bubble can significantly influence the increment in shear layer thickness during reattachment and the resulting blade losses. A frequently used transition length correlation is that of Dhawan and Narasimha[12] developed from studies of constant-pressure flows (see Sec. IV). However, for the case shown in Fig. 1, the application of this correlation with $Re_t = 10^5$ predicts a transition zone over 80% chord in length, which is several times greater than the observed values.

It is this major discrepancy that has prompted the present work. The paper first will review the nature of transition processes on turbomachine blading and existing methods for

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predicting the extent of transitional flow. Finally, a model for the minimum possible length of transitional flow will be introduced and its predictions compared with various experimental results.

II. Transition Processes on Axial Turbomachine Blades

Boundary-layer transition on axial turbomachine blades may result from a variety of causes such as two-dimensional viscous instability, centrifugal instability on concave surfaces, crossflow instabilities, spanwise contamination of turbulent flow from annulus wall boundary layers, diffusion of freestream turbulence into the boundary layer, and surface roughness. It is quite possible for two or more of these phenomena to occur simultaneously and compete for the promotion of turbulent flow.

Laminar instability processes are significantly influenced by external disturbances. The disturbance field in turbomachines is particularly complex and may include 1) pressure fluctuations produced by relative motion between neighboring blade rows, acoustic waves, and shock waves; 2) the passage of wakes from upstream blade rows; and 3) freestream turbulence and entropy fluctuations. It is convenient to separate these disturbances into periodic components (obtained by ensemble averaging) and random fluctuations. The receptivity of the laminar boundary layer to these disturbances may vary throughout the instability process and is influenced by the local boundary-layer Reynolds number and boundary conditions such as streamwise pressure gradient.

For unswept blading of moderate aspect ratio, transition on the convex surface normally will be preceded by two-dimensional viscous instability at low-to-moderate freestream turbulence levels. This is illustrated by Fig. 2, which shows some surface heat-transfer measurements of Turner[4] obtained on a turbine blade in a two-dimensional cascade at Re_c near 10^6. For $Tu = 0.45\%$, there is a sudden jump in heat transfer from laminar to turbulent values around 70–80% chord, indicating a short transition region of about 10% chord and a minor role of freestream disturbances in the transition process. At $Tu = 2.2\%$, the flow behavior is fairly similar, although a slight lengthening of the transition region due to freestream disturbances is evident; but at $Tu = 5.9\%$, there has been a marked expansion of the intermittently turbulent flow region to cover 80% of chord and the influence of freestream turbulence apparently has become dominant. The flow on the concave surface is much more difficult to interpret, since no transition region is discernible and the heat-transfer rates gradually increase from laminar to turbulent boundary-layer values over the whole surface as Tu increases. Similar results have been obtained by a number of other workers.

Figure 1 compares the extent of transitional flow on the convex surface of identical blade sections tested in cascade with $Tu = 0.2\%$ and in a single-stage axial compressor stator. The overall disturbance level in the compressor was around 2% due to the passage of wakes from upstream rotor blades. The transition region on the compressor blade is significantly larger than on the cascade blade, but its mean location is not greatly altered. Walker[5] found that the laminar-turbulent interface on the compressor blade oscillated in a regular manner in sympathy with the rotor wake passage; this transition behavior appeared closely similar to that observed by Obremski and Fejer[6] in oscillating flow over a flat plate.

It may be concluded form the foregoing observations that both periodic and random freestream disturbances can cause a lengthening of the transition region. But it is equally possible for periodic disturbances at a frequency close to that of basic laminar instability processes to shorten the transition region, as will be discussed in Sec. III. This should serve to demonstrate the complexity of the transition process and indicate the fundamental lack of generality of transition correlations based only on disturbance amplitude.

III. Transition Resulting from Two-Dimensional Viscous Instability

The mechanism of laminar-turbulent transition resulting from two-dimensional boundary-layer instability is briefly reviewed here, as it forms the basis for estimates of the minimum transitional flow length to be developed in Sec. V.

This type of transition involves the following sequence of events: 1) a region of instability to two-dimensional disturbances in the laminar boundary layer (Tollmien-Schlichting waves) leading to periodic concentrations of spanwise vorticity, 2) a further three-dimensional instability leading to regular spanwise flow variations and the formation of vortex loops [these may be arranged in a staggered ("thatched") pattern or in streamwise rows], 3) initiation of turbulent spots (or "*breakdown*") through the appearance of high-frequency fluctuations near the heads of the vortex loops, and 4) the final merging of adjacent turbulent spots through both longitudinal and spanwise spreading to form a continuously turbulent flow (a zone of intermittently turbulent flow or "*transition region*"). The spreading of an isolated turbulent spot in

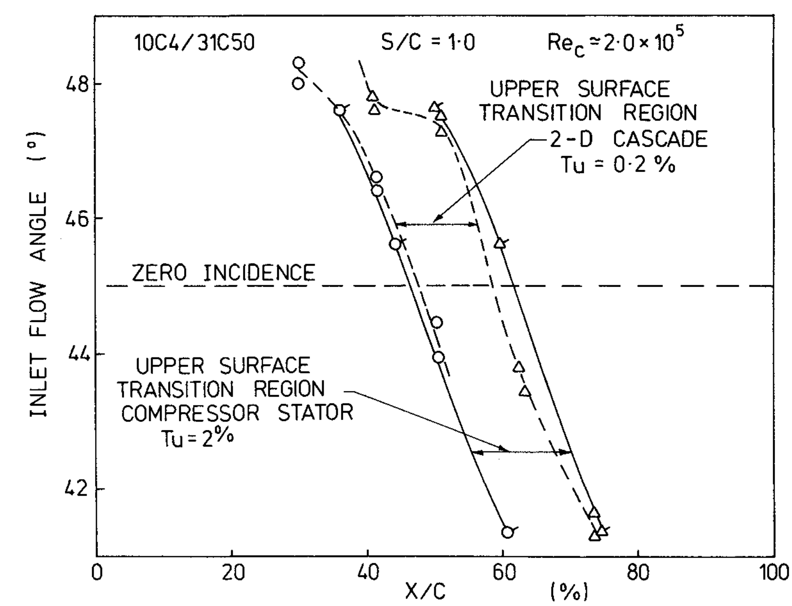

Fig. 1 Extent of transitional flow on similar blading in a compressor cascade[2] and an axial compressor.[25]

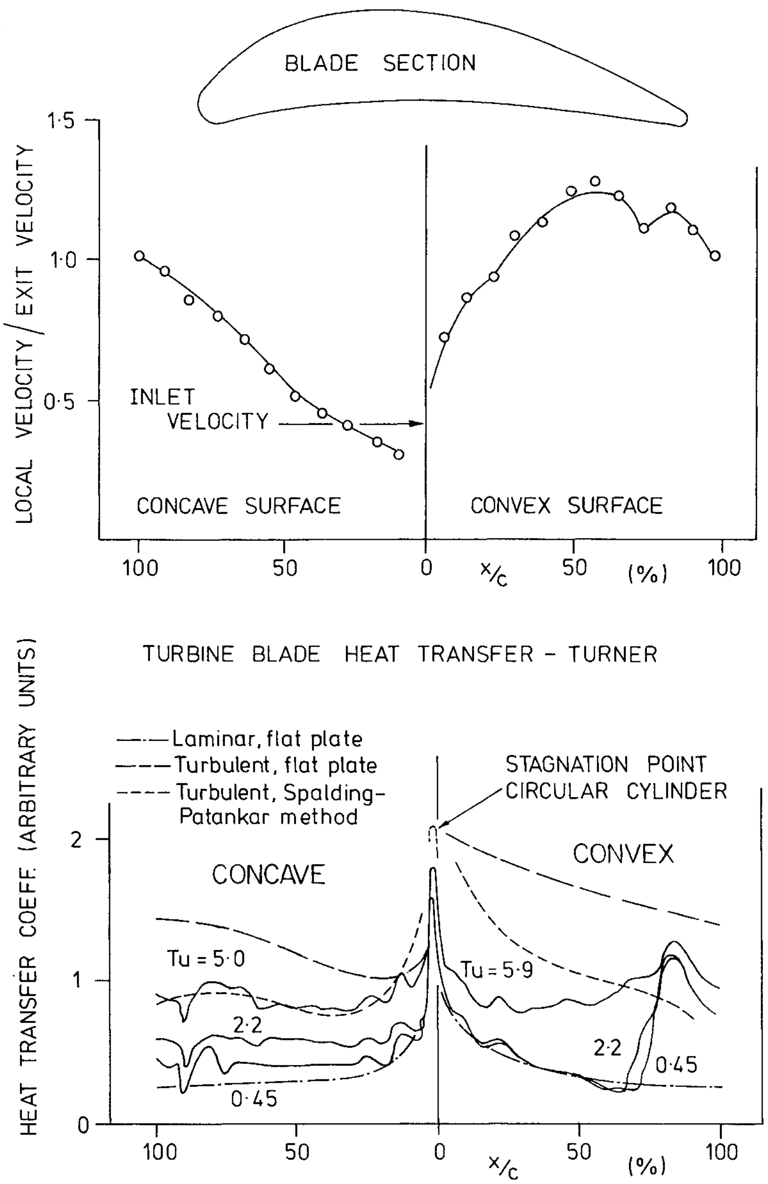

Fig. 2 Experimental distribution of heat-transfer coefficient on an axial turbine blade for exit Mach 0.75 (after Turner[4]).

zero pressure gradient, as observed by Schubauer and Klebanoff,[7] is indicated in Fig. 3.

Studies by Knapp and Roache[8] of transition in zero pressure gradient flows showed that in the absence of significant external forcing by regular external disturbances (i.e., in "*natural transition*") the laminar instability waves broke down in sets. These sets translated several wavelengths of the basic Tollmien-Schlichting (TS) wave during amplification, distortion, and breakdown; they were interspersed by a significant laminar region following breakdown that possibly was caused by local favorable pressure gradients upstream of the breakdown area delaying wave amplification. The vortex loops (or "trusses") resulting from the breakdown of wave sets usually were arranged in a staggered or thatched array. The "intermittency" frequency with which wave sets appeared was about 0.1 times the basic TS wave frequency.

For natural transition in a positive pressure gradient, Knapp and Roache observed a similar behavior to that in zero pressure gradient flow, except that 1) the phenomena became more exaggerated; 2) the formation of wave sets occurred at a higher frequency, and transition approached a continuous process with much shorter intervals between the breakdown of sets; and 3) the thatching tendency of vortex loops before breakdown was greatly reduced, and they tended to appear more in streamwise rows.

In forced transition produced by introducing sound waves with frequencies close to those of the naturally occurring TS waves, Knapp and Roache observed that 1) the frequency of all wave regions became locked into the sound frequency; 2) the transition process in both zero and positive pressure gradients was no longer intermittent (i.e., there was a continuous breakdown of individual instability waves, rather than the intermittent breakdown of wave sets observed in natural transition); and 3) the vortex loops tended to appear in rows rather than in a thatched pattern for both zero and positive pressure gradients.

It is significant to later argument in this paper that the Knapp and Roache observations show two points of physical similarity (i.e., continuous breakdown and streamwise vortex loop arrangement) between the flow processes in forced transition and natural transition in a positive pressure gradient.

IV. Previous Work on Transitional Flow

Constant-Pressure Flows

An extensive review of flow behavior in the laminar-turbulent transition zone recently has been published by Narasimha.[9] Experimental work by Narasimha and others for constant-pressure boundary layers has indicated a universal distribution of turbulent intermittency γ regardless of the agent forcing

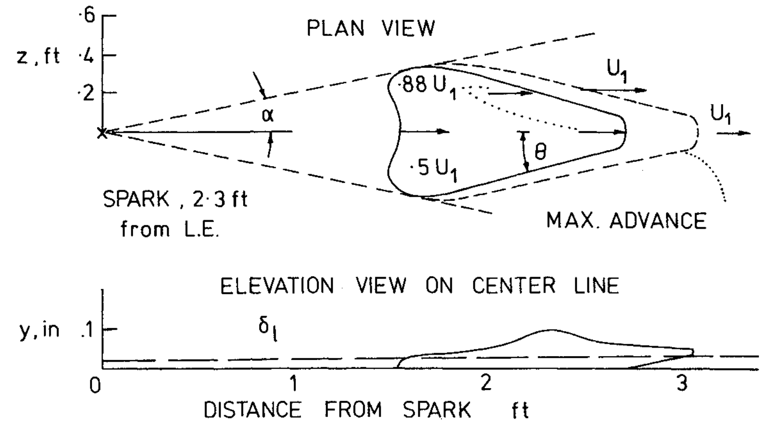

Fig. 3 Development of an artificially generated turbulent spot in zero pressure gradient: $\alpha = 11.3$ deg, $\theta = 15.3$ deg (after Schubauer and Klebanoff[7]).

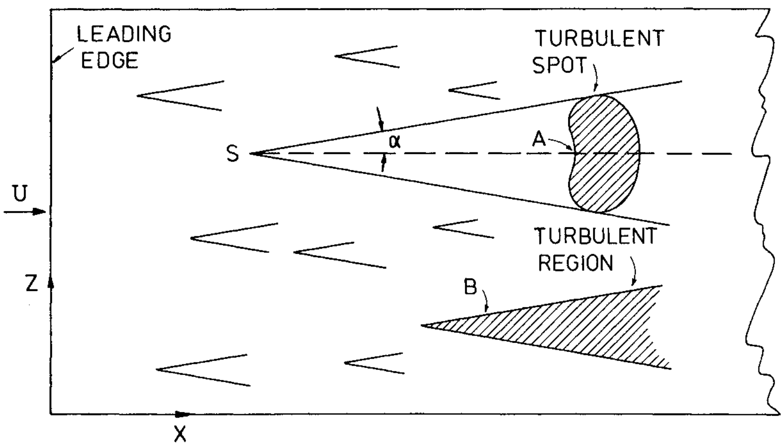

Fig. 4 Transitional flow model of Emmons.[10]

transition. This distribution is described by

$$\gamma = 1 - \exp[-0.412\xi^2] \tag{1}$$

where

$$\xi = (x - x_t)/\lambda \tag{2a}$$

$$x_t = x_{\gamma=0} \tag{2b}$$

$$\lambda = x_{\gamma=0.75} - x_{\gamma=0.25} \tag{3}$$

Defining the downstream limit of the transition zone as

$$x_T = x_{\gamma=0.99} \tag{4}$$

gives

$$L_T = (x_T - x_t) = 3.36\lambda \tag{5}$$

The first major advance in the theoretical description of turbulent flow was Emmons'[10] turbulent spot theory. This envisages transition arising from the growth and coalescence of turbulent spots appearing randomly in both space and time, as shown in Fig. 4. However, Emmons' model of a constant source density for the turbulent spots produced intermittency distributions that did not agree with experiment.

Narasimha[11] later modified Emmons' theory by introducing a concentrated breakdown hypothesis, as shown in Fig. 5. This involves the spots forming randomly in time and cross-stream position at a preferred streamwise location (probably less than $\lambda/3$ in extent) that lies close to the upstream end of the transition zone x_t. The concentrated breakdown hypothesis leads to an adequate model of the intermittency distribution over flow regimes ranging from low-subsonic to hypersonic speeds.

A rather different approach to modeling transition in zero pressure gradient flow was introduced by McCormick.[14] This model assumes that in conditions of low freestream turbulence the turbulent spots appear regularly at the frequency of the basic Tollmien-Schlichting instability wave. Successive spots grow according to an idealized Schubauer-Klebanoff model until they merge to form continuously turbulent flow, as shown in Fig. 6. The random nature of the transition region is allowed for by the random variation of TS wave frequency within the limits allowed by stability theory.

The length of the transition zone in constant-pressure flows has been correlated by various workers in the form

$$Re_\lambda = A\ Re_t^B \qquad (A,B = \text{const}) \tag{6}$$

Dhawan and Narasimha[12] originally proposed

$$Re_\lambda = 5Re_t^{0.8} \tag{7}$$

as an average fit to their available data. As shown in Fig. 7, there was a considerable scatter in this data, with values of Re_λ for a given Re_t varying a factor of 4. Narasimha[13] later revised this correlation to give

$$Re_\lambda = 9Re_t^{0.75} \tag{8}$$

which is still consistent with the Dhawan-Narasimha[12] data and is more physically appealing in that the turbulent spot breakdown rate becomes mainly dependent on local boundary-layer thickness.

Transition in a Pressure Gradient

Chen and Thyson[15] modeled the transition zone for flows in a pressure gradient by assuming that 1) spot propagation

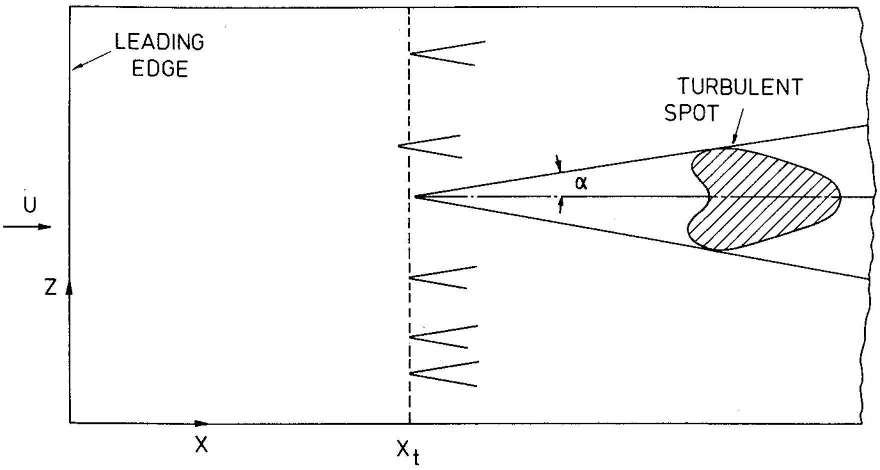

Fig. 5 Transitional flow model of Narasimha.[11]

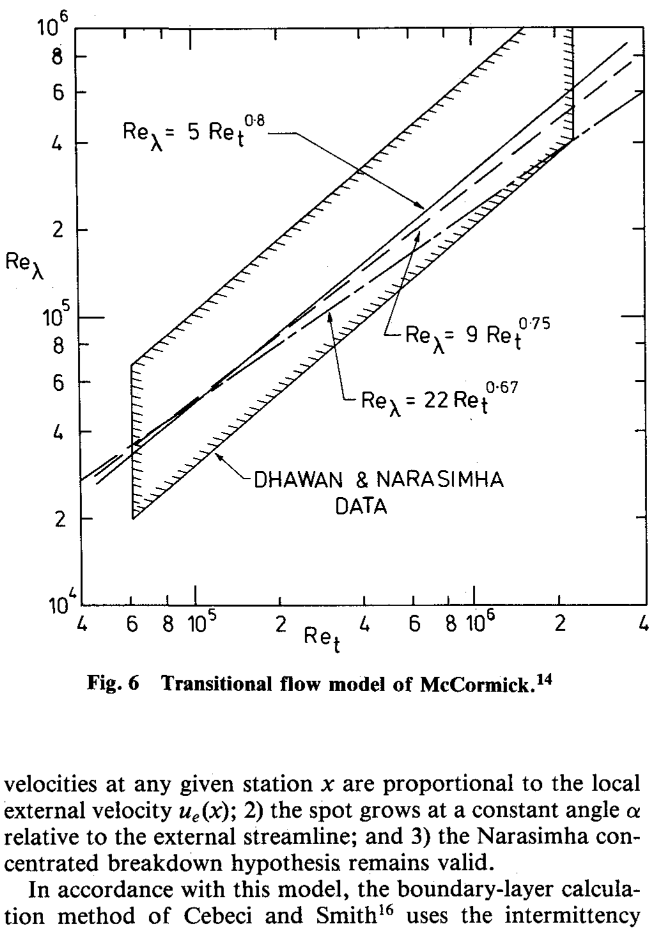

Fig. 6 Transitional flow model of McCormick.[14]

velocities at any given station x are proportional to the local external velocity $u_e(x)$; 2) the spot grows at a constant angle α relative to the external streamline; and 3) the Narasimha concentrated breakdown hypothesis remains valid.

In accordance with this model, the boundary-layer calculation method of Cebeci and Smith[16] uses the intermittency distribution

$$\gamma = 1 - \exp\left[-G(x - x_t)\int_{x_t}^{x}\left(\frac{\mathrm{d}x}{u_e}\right)\right] \tag{9}$$

where G is a transition length parameter given by

$$G = (1/1200)(u_e^3/\nu^2)Re_t^{-1.34} \tag{10}$$

For the case of zero pressure gradient, this reduces to

$$Re_\lambda = 22Re_t^{0.67} \tag{11}$$

which gives similar results to Eqs. (7) and (8) due to compensating variations in the constants used. However, as shown by Fig. 7, Eq. (11) tends to predict rather lower transition lengths for $Re_t > 10^6$.

More recent work reviewed by Narasimha[9] has indicated that the presence of pressure gradients can cause marked deviations from the universal intermittency distribution for constant-pressure flows. The more complex spot propagation characteristics in the presence of pressure gradients appear related to associated variations in stability of the laminar boundary layer. The strongest influence occurs in favorable pressure gradients, which can cause a reduction in spreading angle and inhibit the longitudinal spreading rate of turbulent spots. Doorly and Oldfield[17] observed leading- and trailing-edge velocities of 0.8 and 0.6 times the local freestream velocity for an isolated turbulent patch (generated impulsively at the airfoil leading edge by shock wave passage) in accelerating flow over a turbine blade.

Transition from Diffusion of Freestream Turbulence

A transition mechanism that bypasses the laminar instability process completely is the diffusion of turbulent energy from the freestream into the boundary layer. McDonald and Fish[18] modeled this process by using a turbulent boundary-layer calculation method with a suitable initial distribution of turbulent energy across the layer at the upstream end of the transition region. A more recent study of this type was carried out by Rodi and Scheuerer,[19] who found that freestream turbulence levels greater than about 1% were needed to initiate the transition process; their model had a tendency to predict a transition length L_T somewhat shorter than observed experimentally.

Both Hodson[3] and Doorly and Oldfield[17] have speculated that passing wakes could induce transition on axial turbomachine blades through this process. However, the available data are rather limited, and the interpretation of observations is complicated by the simultaneous occurrence of periodic freestream oscillations associated with the wake passage.

V. Model for Minimum Length of the Transition Zone

A model for the minimum possible length of transitional flow now will be developed. It is assumed that this is achieved by forcing of the laminar instability process, with regular inception of an equispaced spanwise array of turbulent spots from x_t once each cycle of the primary TS wave. McCormick's[14] model for the spreading of turbulent spots then is applied to determine the streamwise distance required for the first meeting of adjacent spots. The latter distance is used as an approximate transition length, although it must clearly underestimate L_T to some extent. Lateral and longitudinal spreading of turbulent spots are examined separately to see which case is more limiting.

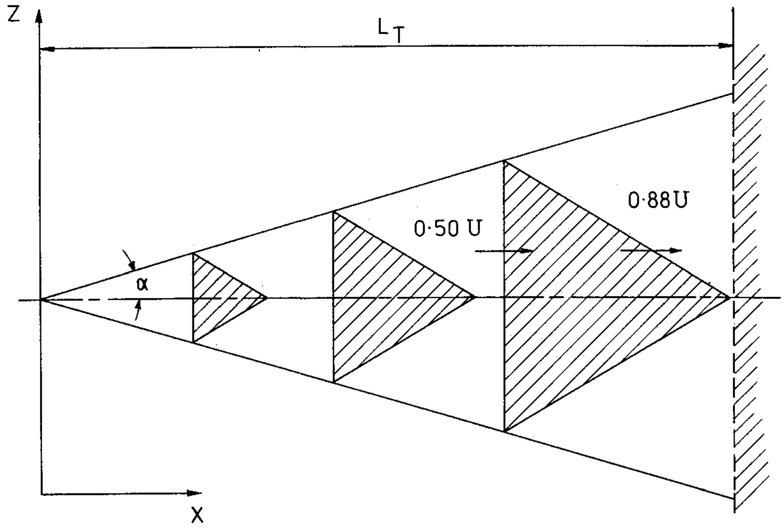

Fig. 7 Comparison of transition length correlations for zero pressure gradient.

Longitudinal Merging of Turbulent Spots

It is first necessary to determine the frequency of spot inception. Numerous studies have shown the dominant disturbance frequency at breakdown to be well predicted by the TS wave frequency having the maximum amplification ratio according to small-disturbance theory. Figure 8 shows the results of stability calculations for Falkner-Skan profiles obtained by Obremski et al.[20] It is seen that the locus of maximum amplification *rate* is well described over the whole range of pressure gradient parameter β by

$$(\omega\nu/u_e^2) = 3.2Re_{\delta^*}^{-3/2} \tag{12}$$

Equation (12) then is assumed to approximate the frequency of disturbances having the maximum amplification *ratio*. The corresponding disturbance period is

$$T = 2.0(\nu/u_e^2)Re_{\delta^*}^{3/2} \tag{13}$$

Applying McCormick's idealized model of turbulent spot geometry as shown in Fig. 6, together with the spot propagation velocities of Schubauer and Klebanoff for zero pressure gradient flow, the streamwise distance for the first longitudinal meeting of adjacent spots on the spot center lines gives

$$L_T = 1.16u_eT \tag{14}$$

Combining Eqs. (13) and (14) leads to

$$Re_{L_T} = u_eL_T/\nu = 2.30Re_{\delta^*_t}^{3/2} \tag{15}$$

Finally, substituting

$$Re_{\delta^*_t} = 1.72Re_t^{1/2} \tag{16}$$

for zero pressure gradient flow, Eq. (15) can be written as

$$Re_{L_T} = 5.20Re_t^{3/4} \tag{17}$$

Lateral Merging of Turbulent Spots

Recent theoretical studies of three-dimensional laminar instability (e.g., Orszag and Patera[21]), suggest that the spanwise wavelength of three-dimensional disturbances is approximately equal to that of the primary two-dimensional TS waves at breakdown. This conclusion is supported by the experimental studies of forced transition carried out by Schubauer and Skramstad[22] and Kegelman and Mueller.[23]

Using Eq. (13) for the period of the TS waves, in combination with their phase velocity C_r, an approximate value for the TS wavelength is

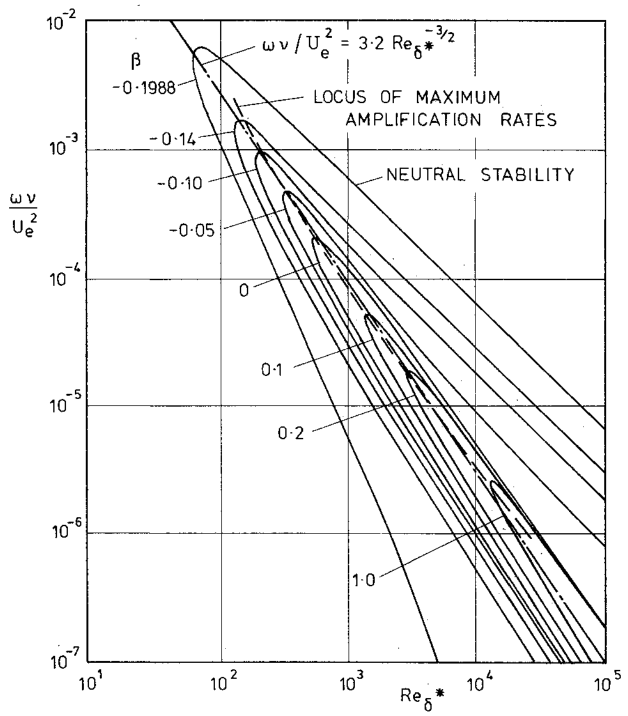

Fig. 8 Stability characteristics of Falkner-Skan laminar boundary-layer profiles calculated by Obremski et al.[20]

$$\Lambda_x = 2.0(C_r/u_e)(\nu/u_e)Re_{\delta^*}^{3/2} \tag{18}$$

Assuming spot inception occurs from x_t at regular spanwise intervals equal to this value, together with a spreading angle α of 10 deg on each side, u_e constant leads to a streamwise distance for the first lateral touching of adjacent spots given by

$$L_T = 5.6(C_r/u_e)(\nu/u_e)Re_{\delta^*}^{3/2} \tag{19}$$

(or roughly three wavelengths of the primary TS wave) so that

$$Re_{L_T} = 5.6(C_r/u_e)Re_{\delta^*}^{3/2} \tag{20}$$

This gives the same functional dependence on Re_{δ^*} as Eq. (15) for longitudinal spot merging. For zero pressure gradient flow, the factor $5.6(C_r/u_e)$ falls from about 2.2 at $Re_{\delta^*} = 600$

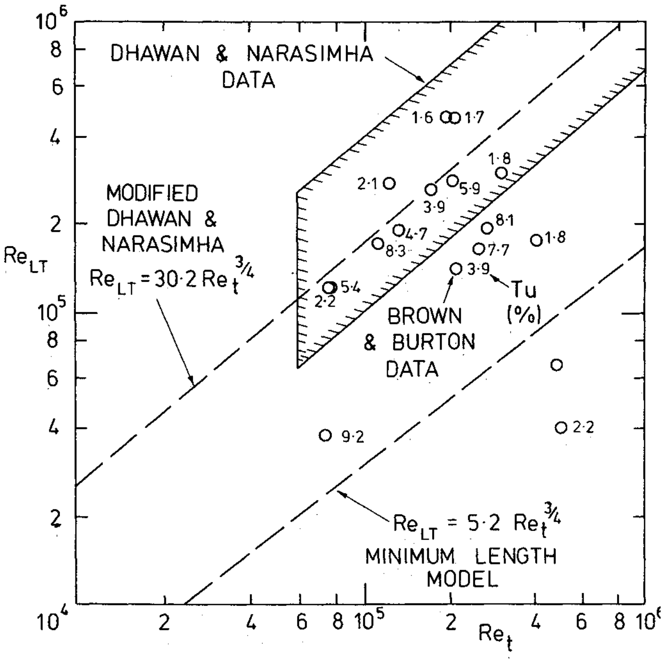

Fig. 9 Comparison of transition length correlations with experimental data for arbitrary pressure gradient.

to 1.2 at $Re_{\delta^*} = 10{,}000$. Comparison with the factor of 2.30 in Eq. (15) indicates that the streamwise distance for lateral spot merging generally will be less than that for longitudinal merging.

Comparison with Experiment: Zero Pressure Gradient

For zero pressure gradient, the streamwise distance needed to complete transition will be controlled by longitudinal spot merging for $Re > 10^5$, and the minimum transition length therefore is taken to be that given by Eq. (17). Recasting Narasimha's empirical correlation of transition length [Eq. (8)] in terms of L_T using Eq. (5) leads to

$$Re_{L_T} = 30.2 Re_{\delta^*}^{3/4} \tag{21}$$

which shows the same functional dependence on Re_t as Eq. (17) but gives a value of Re_{L_T} around six times greater. Although Eq. (17) somewhat underestimates L_T, this cannot possibly account for such a huge discrepancy. The error due to intermittency being less than 100% when spots first touch longitudinally is rather less than indicated by Fig. 6, since the spot boundaries actually lie outside the idealized triangular planform and prior lateral merging will further tend to increase the intermittency.

Much better agreement is obtained with the observations of transition resulting from forced oscillations by Schubauer and Skramstad.[22] For the case shown in Fig. 36 of Ref. 22, the chosen forcing frequency is almost identical to that obtained from Eq. (12) of this paper; this eliminates one potential source of error. The first turbulent spots appear at 32 in. from the vibrating ribbon introducing the disturbance and 6 in. further downstream (corresponding to $\Delta Re \simeq 155{,}000$) the intermittency appears to be around 90%; adopting this value of γ [corresponding to $(x - x_t)/\lambda \simeq 2.5$] would give $Re_{L_T} \simeq 210{,}000$ (corresponding to $L_T/\lambda = 3.36$). By comparison, Eqs. (15) or (17) derived from longitudinal spot merging gives $Re_{L_T} \simeq 250{,}000$, while Eq. (20) derived from lateral merging leads to $Re_{L_T} \simeq 185{,}000$.

Clearly, the present model gives transition lengths of the right order for the forced transition case. It is perhaps surprising that Eqs. (15) and (17) do not somewhat underestimate L_T as had earlier been expected. It may be that the dynamics of individual spots in their initial stages of development differ slightly from those of the well-developed spots that have been studied experimentally. Schubauer and Klebanoff's observations of spreading rates, for example, were carried out well downstream of the initiation points where the spots would have covered many Tollmien-Schlichting wavelengths in the spanwise direction.

Minimum Transition Length: Arbitrary Pressure Gradient

In order to determine the minimum length of transitional flow on airfoils, it is necessary to apply the present model in conditions of arbitrary pressure gradient. This is quite straightforward, since Eq. (12) specifies the dominant disturbance frequency in terms of local boundary-layer Reynolds number and is valid for any pressure gradient. It only remains to allow for the variation of spot propagation velocities with x, and this can be done by applying the Chen-Thyson model[15] discussed in Sec. IV. In most cases of practical interest, the transition length will be relatively short, and it will suffice to use Eq. (15) with an average value of u_e over the transition zone.

The model should give reasonable predictions for positive pressure gradients, in which case the natural transition behavior more closely resembles the continuous wave breakdown of forced transition. Poorer agreement must be expected in accelerating flows, where transition behavior will tend toward wave breakdown in sets, and there is greater uncertainty about spot spreading rates. The transition length predicted by Eq. (15) is likely to be an underestimate in the accelerating flow situation, but this does not detract from its value in estimating a minimum value of L_T.

Comparison with Experiment: Turbomachine Blade Flows

The above model is first tested against the compressor cascade data shown in Fig. 1. Using the value of Re_{δ^*} obtained from measurements on the compressor stator, where the flow is known to be closely similar, the transition length at 1 deg incidence is found to be 22% of chord. This is about double the observed value of L_T but is in far better agreement than the value of around 85% chord given by Narasimha's correlation [Eq. (22)]. The transition data presented in Fig. 1 are based on qualitative observations, rather than quantitative intermittency measurements, and there therefore could be a tendency for the experimental values of L_T to be underestimated.

Figure 9 shows a plot of transition length data for various turbine blade flows with arbitrary pressure gradient and different freestream turbulence levels as presented by Brown and Burton.[24] The modified Dhawan-Narasimha correlation [Eq. (21)] and the present minimum transition length model [Eq. (17)] are indicated for comparison. There is a tendency for the turbine blade data to lie between these two relations.

VI. Conclusions

It has been shown that transition length correlations developed from constant-pressure flows may seriously overestimate the length of transitional flow in a positive pressure gradient situation. A physical explanation for this has been proposed in terms of the tendency for natural transition in zero pressure gradient to evolve from the breakdown of laminar instability waves in sets interspersed by regions of stable laminar flow. The transition process in a positive pressure gradient is more akin to that in forced transition where there is a continuous breakdown of individual instability waves into turbulence and the length of intermittently turbulent flow is considerably shorter.

A minimum transition length model has been developed from a continuous breakdown hypothesis. This closely approximates the forced transition situation and should give reasonable estimates of transition length for boundary layers in a positive pressure gradient with low-to-moderate freestream turbulence levels. In zero or negative pressure gradients, where the natural transition process will tend to dominate, the present model will underestimate the transition length.

High levels of freestream turbulence and periodic disturbances or large scale (such as passing wakes) also may lengthen the transition region. The promotion of earlier transition onset and a delayed completion of transition both may contribute to this lengthening. These effects are difficult to quantify because they depend on a wide variety of factors such as local boundary-layer parameters and the global flow pattern around a body. For example, several workers have observed the wake chopping phenomenon to initiate a turbulent flow wedge at the leading edge of an axial turbomachine blade, with a consequent marked lengthening of the transition region on the blade surface. For the case shown in Fig. 1, however, the wake-chopping phenomenon did not initiate turbulent flow at the leading edge; the transition length on the compressor blade was only slightly longer than that on a similar section tested in a low-turbulence cascade.

In summary, the transition length correlation given by Eq. (15) should provide an estimate for the minimum possible length of intermittently turbulent flow, which will be preferable to using a point transition model. This is especially true for flow around axial turbomachine blading where, because of the low Reynolds number operation, the transitional flow occupies a relatively greater proportion of the blade chord. In engineering flow situations, the actual transition length probably will lie somewhere between the values predicted by the present model and those of zero pressure gradient correlations. These two limiting values should be useful in sensitivity analyses of airfoil flow computations.

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